At long last, this brings us to....

(V.) Random Coupling Model (Randomly Phased Coupling)

→ consider particular choice of ϕ's, as having random phases;

$$\phi_{\alpha,\beta,\alpha-\beta} = e^{i\theta_{\alpha,\beta,\alpha-\beta}}, \quad \theta \text{ real and:}$$

$$\theta_{\alpha,\beta,\alpha-\beta} = -\theta_{\alpha,\beta,\alpha-\beta}.$$

→ let, for each α, β, θ be random on $[0, 2\pi]$

→ $|\phi|^2 = 1$, but ϕ has random phase

→ $\neq$ RPA; here ϕ, not q, has random phase.

⇒ random coupling model.

Now, consider C's:

$\rightarrow$ $$C_{2,1} = \phi_{\alpha,\beta,\alpha-\beta}\,\phi_{\alpha-\beta,-\beta,\alpha}$$
$$= e^{i\theta} e^{-i\theta} = 1$$

$\rightarrow$ $$C_{4,3} = \frac{1}{M}\sum_{\gamma} \phi_{\alpha,\beta,\alpha-\beta}\,\phi_{\alpha-\beta,\gamma,\alpha-\beta-\gamma}\,\phi_{\alpha-\beta-\gamma,-\beta,\alpha-\gamma}\,\phi_{\alpha-\gamma,-\gamma,\alpha}$$
$$= \frac{1}{M}\sum_{\alpha} e^{i\theta_1} e^{i\theta_2} e^{i\theta_3} e^{i\theta_4}$$

where θ_1, θ_2, θ_3, θ_4 random, so

$$C_{4,3} \sim O(1/M)$$

so $\rightarrow$ $C^{\text{irred.}}_{2n,p} \xrightarrow[M\to\infty]{} 0$ for all $n > 1$

$\rightarrow$ only survivors are conjugate pair terms.

so, propagator evolution equation reduces to:

$$\frac{dG}{dt} + \langle b^2 \rangle G * G = 0, \qquad G(0) = 1$$

$$\frac{dG}{dt} + \langle b^2 \rangle \int_0^t G(t-s) G(s)\, ds = 0$$

$b_* = b_{rms}$

$$\Rightarrow \left[\begin{array}{l} G(t) = J_1(2 b_* t) / b_* t \\ G(\omega) = (\pi b_*)^{-1} \left[1 - (\omega / 2b_*)^2 \right]^{1/2} \quad (|\omega| < 2b_*) \\ \quad\;\; = 0 \quad (\omega > 2b_*) \end{array} \right.$$

$\Rightarrow$ DIA propagator is exact solution (for $M \to \infty$) of Random Coupling Model

Thus:

$\rightarrow$ Random Coupling Model (with $M \to \infty$) is realization of DIA

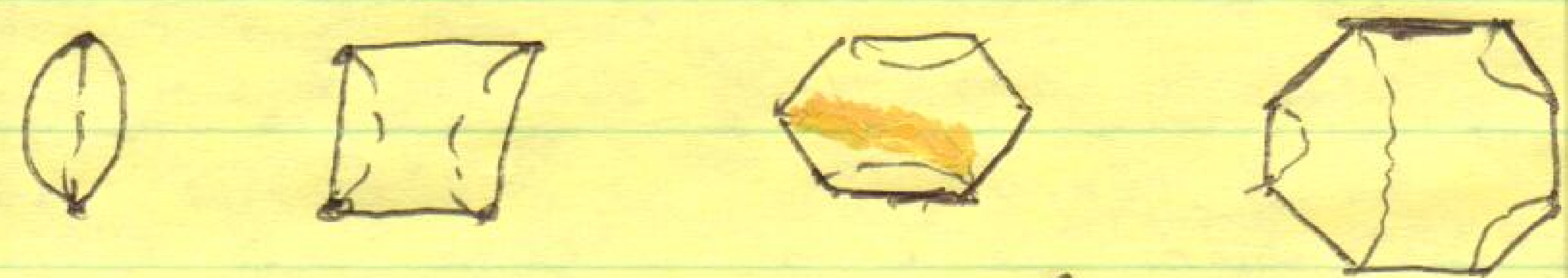
$\rightarrow$ DIA is shown realizable (i.e. corresponds to physical model/system ...).

→ So, after all this, what does it mean?

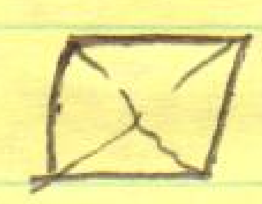
- Renormalization a la' D.I.A. is a partial resummation of an infinite number of diagrams

i.e. $\sum_{n,p} C_{2n,p}$ retains reducible diagrams of form $\sim (C_{2,1})^n$
neglects others

∴

i.e. retained:  , etc.

→ all factorizable to $(C_{2,1})^n$

neglected (arbitrarily):  , etc.

→ h.o. irreducible

Thus - D.I.A. does retain contributions from "all orders" → some "hint" at strong coupling
- D.I.A. does not do so:
→ uniquely
→ systematically

uniqueness: ladder diagrams (other problem)

systematics: no way to estimate what is neglected.

insight? : no simple 'physical feel' for neglected diagrams.

∴ D.I.A. is useful, albeit uncontrolled, approximation

- Why Direct Interaction Approx?

→ $\langle G_{\alpha\alpha} \rangle$ → response fctn. decay due energy transfer

→ decay / transfer ⟺ phase relation for triad must be established

→ route for decay

- direct: $q_\alpha \, q_{\alpha-\beta}$ via $\Phi_{\alpha,\beta\alpha+\beta} \, b_\beta$ → survivor in random coupling approx.
- indirect: via background mode 'chains' of intermediaries

→ closing of planar diagram on itself gives "reaction" of couplings back on α ⟺ damping of q_α.

→ G factors → relax the phase relation

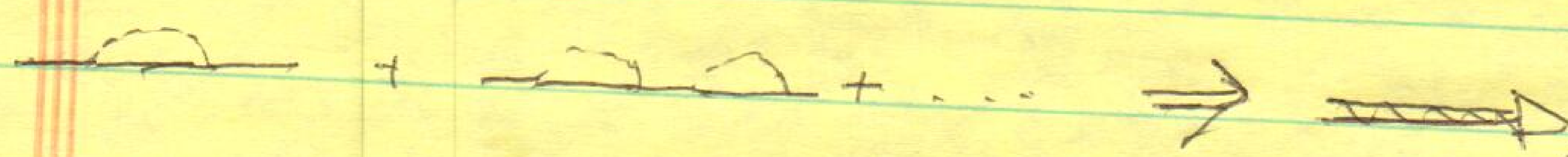
↓ convolution expansion

longer 'chains' ⟺ more factors

→ in random coupling → only **direct** interaction transfer retained, à la $C_{3,1}$

⇒ **Direct** interaction approximation

→ random coupling approximates:



akin $\left\{ \begin{matrix} \text{mass} \\ \text{propagator} \end{matrix} \right\}$ renorm.

Key question: Can one get/develop some **physical** insight for some systematic class of diagrams entering G **but** neglected by D.I.A./random coupling?